



MathILy 2021 Final Report

Preface

MathILy was again different from usual this year—because of the continued global pandemic, the program was again held synchronously online. We still managed to hold a program with excellent students who learned a lot of mathematics and a lot about how to think and speak and write mathematically.

Program Preparations

Promotions

Electronic: Individual emails were sent to prior participants and promising applicants. MathILy continues to be listed on several high-traffic webpages, including MIT Admissions and AMS summer program listings. Web traffic varied from 6000–10000 hits/month, with about a third of the traffic from abroad. The second-most-visited page was Discrete Mathematics in the Real World, indicating that this is effective outreach.

Print: None, as most contests were either cancelled or happened virtually.

Other Activities: We held a {MathILy, MathILy-Er, MathILy-EST} Yearly Gather at the Joint Mathematics Meetings (online) where the 45 participants, about 2/3 of whom were alumni, worked through Jonah-Ostroff-designed three-player escape rooms in Google Sheets. In April, sarah-marie offered an Art of Problem Solving Math Jam (80–100 attendees at any time) on multibackwards numbers followed by a {MathILy, MathILy-Er} Q&A. HMMT was online with no opportunities for mini-events, and similarly ARML was online with no Friday Night Lecture.

Applications and Admissions

Statistics: We received 1354 Short Forms, 491 Not-as-Short Forms, 444 EARs, and 378 completed applications. We admitted 62 students; thus, our current admissions rate is roughly 16%. Nineteen students declined, 8 for 7 other summer mathematics programs. We routinely ask attending students about their first choice of summer program. This year was by far a record, with 26 of the 43 attending students saying MathILy was their first choice.

Demographics: Applicants originated from at least 39 US states/territories/districts and 28 foreign countries (representing mostly Europe and Asia, but also including South America, Oceania, and the Middle East). The data in the following table was measured where possible and approximated otherwise; the final row reflects a post-program demographic survey given as part of assessment for the MathILy-EST NSF grant.

Percentage	Female	NB	East Asian	South Asian	Latinx	Middle Eastern	Other of color
Short Forms	40%	1%	34%	24%	4%	15%	2%
EARs	38%	1%	47%	20%	3%	3%	1%
Attending	40%	7%	51%	19%	2%	0%	0%

Financial Aid: We awarded \$1,650 in financial aid to MathILy participants (all to one international student), and used our \$2,520 AMS Epsilon Fund grant for this purpose. Only 3% of admitted students applied for financial aid; we met the demonstrated need of all applicants.

Personnel

Academic: Lead Instructors were Dr. Hannah Alpert (Auburn University, Ph.D. MIT 2016), dr. sarah-marie belcastro (Math Staircase Inc., Ph.D. U. of Michigan 1997), Dr. Max Engelstein (U. of Minnesota, Ph.D. U. of Chicago 2016), Dr. Brian Freidin (Auburn University, Ph.D. Brown U. 2018), Dr. Thomas C. Hull (Western New England U., Ph.D. Univ. of Rhode Island 1997), Dr. Emily Peters (Loyola U. Chicago, Ph.D. UC Berkeley 2009) and Dr. Daniel Studenmund (Binghamton U., Ph.D. U. of Chicago 2014). Apprentice Instructors were Nadav Kohen (graduate student at Indiana University and MathILy 2015 alum), Joshua Munding (graduate student at U. of Chicago and MathILy 2013 alum), Kye Shi (undergraduate at Harvey Mudd and MathILy 2015/2016 alum), Natasha Ter-Saakov (graduate student at Rutgers U. and MathILy 2014/2015 alum) and Corrine Yap (graduate student at Rutgers U.). The FACToR (Facilitator of Activities and CriTiquer of wRiting) was Allen Wang (undergraduate at MIT and MathILy 2016/2018 alum).

Biographical information and prior experience are listed at [Dramatis Personae](#).

Administrative: The Director was dr. sarah-marie belcastro. The excellent {MathILy, MathILy-Er} Minion was Madison Stuart (Smith College B.A. 2006 in math and German; graduate work in information science at the University of Michigan). The PRiMEs (Protectors and Responders in the MathILy Environment) were Johnna Farnham (bachelor's in math, Western New England U. 2021) and Patrick Chan (bachelor's in math, Loyola U. Chicago 2021).

Advisory Amalgam: These individuals gave advice on academic and practical aspects of MathILy.

[Dr. Douglas J. Shaw](#), mathematics faculty at University of Northern Iowa

[Dr. Ruth Haas](#), mathematics faculty at University of Hawaii

[James Cocoros](#), mathematics faculty at Stuyvesant High School

[Dr. Dylan Shepardson](#), mathematics faculty at Mount Holyoke College

[Dr. Carol E. Fan](#), operations researcher (currently Operations Data Science Lead at Apple)

[Dan Zaharopol](#), Executive Director of [BEAM](#)

[Dr. James Tanton](#), mathematician, currently Ambassador for the [MAA](#)

[Dr. Joshua Greene](#), mathematics faculty at Boston College

[Dr. Emily Peters](#), mathematics faculty at Loyola University Chicago

[Wing L. Mui](#), Seattle-area artist and former mathematics teacher

[Dr. Thomas Hull](#), mathematics faculty at Western New England University

[Dr. Josh Laison](#), mathematics faculty at Willamette University

Student Demographics

U.S. States represented by MathILy students, roughly from east to west: Massachusetts, New York, New Jersey, Pennsylvania, Ohio, Illinois, Texas, California, Washington, Oregon, and Alaska.

Countries outside of the United States represented by MathILy students, roughly from east to west: China, India, United Arab Emirates, Belgium, Brazil, Canada.

Gender breakdown: There were 17 female, 3 nonbinary, and 23 male participants.

Ages: There were six 14-year olds, ten 15-year olds, twenty 16-year olds, six 17-year olds, and one 18-year old.

Academic backgrounds: About 45% of the students had already taken calculus II or equivalent (and 14% had also taken multivariable calculus), two had taken linear algebra, four had taken some number theory, and one had taken topology. In contrast, 5% of the students had not yet taken precalculus. 20 students had attended summer mathematics programs before.

What Happened at MathILy 2021?

Academics

Classes: Classes met through Zoom for audio/video communication, with breakout rooms for group work; we also used Limnu shared online whiteboards, Google Drive for sharing class notes, Slack for during-class and outside-of-class communication, Google Classroom for submission/return of written work, Kami for commenting on written work, our usual computer algebra systems Sage and *Mathematica*, optional LaTeX for typesetting, and gather.town for inter(re)views.

Each weekday we had 4 hours of morning class, 1–1.5 hours of Daily Gather, and 3 hours of evening class, for at least 8 contact hours per day. The general Saturday template consisted of 4 hours of morning class and 2 hours of afternoon Life Seminar, and we abstained from formally scheduled activities on Sundays so students would have a full non-Zoom day.

The basic curricular structure was two weeks of core curriculum, called Root Class (after the root of a graph theoretic tree, and after the idea that the material strengthens student grounding much as the roots of a tree do), followed by one week of short topical classes, called Week of Chaos, followed by two weeks of focused-topic classes, called Branch Class (after branches of mathematics, and after the idea that tree branches grow from a strong trunk nourished by roots).

Root Class: There were three Root classes, each with 14 or 15 students, one with {sarah-marie, Kye, Johnna}, one with {Tom, Corrine, Patrick}, and one with {Brian, Josh, Natasha}. Our core curriculum consisted of linear and affine algebra and geometry (including equations and intersections of hyperplanes, span, linear independence, transformations, and dimension), combinatorics, graph theory, definition and examples of groups, isomorphism for various categories, probability spaces and expected value, and basic cardinality. Of course, all of this material was treated with full proofs given by the students.

Week of Chaos: Students indicated which of 55 potential topics they would be excited to learn about, from which instructors decided on a list of 28 classes offered. These were: Demented Dimension, Information Theory, Twitch Plays Minecraft... (random walks), Fibontities: Return of the Twordered Sums, What Shape is Best? (isoperimetric problems), Sneaky Proofs (zero-knowledge proofs), Origametry, Knot Theory!, Apocalypse Planning (with Levity) (algorithms), Cookie Sets (Julia sets), Erdős Magic (probabilistic method), Fancy Note-Taking Diagrams (sorting algorithms), Counting with Dumplings and Muffins (linear algebra methods in combinatorics), Loops on Loops on Loops (homotopy theory), Analyzing my Childhood Best Friend (complex analysis), Math Saves the World: An Introduction to Infectious Disease Modeling, Keeping Secrets, Game Theory, Generatingfunctionology, Math Saves the World: Combinatorial Optimization, Artistic Anthropods (graph coloring), Class of Chaos (and fractals), Totally Pristine Algebra (Cardano's formula for solving a cubic), Voting theory, or Why We Can't Have Nice Things, Möbius Transformations Revealed, Somewhat Dirty Algebra (groups and rings), What norms can do for you (p -adics), and Too Much Slack Syndrome (Markov chains). Student preferences guided placement of each student into 5 classes. More than half of these classes used specific material from the Root curriculum, a bit more than 1/4 benefitted substantially from students' knowledge/understanding of linear algebra, and almost 1/5 used computer algebra systems in class.

Branch Classes: We offered three Branch classes, one on topological graph theory (sarah-marie and Nadav), one on persistent homology (Hannah, Josh, and Kye), and one on discrete geometry (Tom and Natasha with Johnna). All three Branch classes used linear algebra and all introduced applications of the material.

Pedagogy: All classes were conducted using inquiry-based learning, with the bulk of the time spent with students working in groups or presenting their insights to each other and a much smaller amount of time used by faculty conducting discussion from the shared whiteboards.

Feedback: Students received feedback in multiple ways. During class, they received instant verbal feedback on the correctness of their mathematical ideas, and also on use of notation, language, and presentation style. Likewise, students received daily written feedback on their mathematical writing. Near the end of Root and Branch classes, each student was asked to write an introspective self-evaluation. The self-evaluations were discussed by the student's instructors, and the instructors then held a 5–20 minute meeting with each student to give overall feedback on the student's progress at MathILy and advice for the future.

Interactions with MathILy-EST: MathILy-EST participants intermixed with MathILy students for group work in Daily Gathers throughout the program. Additionally, each MathILy-EST participant took a Week of Chaos class.

Daily Gathers: Each instructor gave a Daily Gather, as did the MathILy-EST research group. The Daily Gather timeslot was also used to show Math Movies once per week. These included expository films made by the Mathematical Association for America, films from the National Film Board of Canada, and narrated animations made by individual mathematicians or research groups. The remainder of the Daily Gathers were given by visiting mathematicians. Each Daily Gather speaker provide some insight into that person's perspective on the mathematical enterprise and/or way of being a mathematician.

Extracurriculars

Life Seminars: There were four weekend Life Seminars offered, all with time for open questions. The first was on practical matters such as how to address faculty (in person or by email) and impostor syndrome. The second Life Seminar was on careers for people with mathematical science training, for which we had guests from industry. The third Life Seminar was about preparing for Branch and when to start research. The final Life Seminar was on how to choose colleges to which to apply, and included advice from MathILy-EST participants as well as an exchange of information about instructor experiences at colleges and universities.

Other social activities: Before the opening meeting we had four hours of structured socializing (to mimic students hanging out in a common room) involving gather.town exploration, Two Truths and a Lie, Trivial Debates, and Math-ictionary simultaneous with Codenames. At the end of the first Life Seminar, we played telephone pictionary in teams using Drawception. Throughout the program students played games in the afternoon. A staff member hosted a Sunday-morning breakfast Zoom to which many participants brought the traditional chocolate-chip muffins. Most nights there were optional graphic-novel Bedtime Stories.

During Week of Chaos, students started doing math together nightly after Bedtime Stories, including using a private #woc-nonsense Slack channel and "Pin Irresponsible" on the Complex Analysis limnu board. The program toured Bryn Mawr on Saturday morning right after Week of Chaos, using first Google Earth and Google Maps. That afternoon, we held some in-person meetups, in Boston, San Francisco, Chicago, and Philadelphia. The following day some students and staff celebrated National Ice Cream Day in gather.town. One Branch class was obsessed with pairs of pants and another regularly featured sugared tori.

Administrative matters

Student early/late time zones: We held activities in Central time because it was closest to average; this resulted in four students in very different time zones ranging from 3 hours early to 13 hours later.

Technology selection and preparation: After reviewing available options, we used the same nine pieces of software this summer as we did last summer (Zoom, Limnu, Slack, Google Drive, Google Classroom, Kami, LaTeX, Sage, and *Mathematica*) plus one new piece: gather.town. This site allows an account holder to design a floor plan on which can be placed interactive elements, and where authorized users can navigate avatars such that avatars near each other on the floor plan automatically share audio and video.

We revised our prior documentation for each software, and staff then did a tech check with each student to uncover and resolve unexpected technological issues.

Post-Processing

Post-program meetings: After the official end of the program, the staff convened to evaluate various aspects of the program and to discuss how we could improve the workings of MathILy in future. We began by assessing our attempts to streamline time and energy expenditure for online teaching, and concluded that some of these will also be useful when we are again in person. An innovation this year that will be carried forward is a way to track invented terminology. Later meetings generated a substantially faster way to place students in Week of Chaos classes, and plans for teaching discussions run by experienced AIs for newer AIs.

Impact: As usual, many students commented that they learned about areas of math of which they hadn't even dreamed before MathILy, and that they grew substantially in their proofwriting prowess. Several also commented that they gained new perspectives on mathematics. Some said that MathILy rekindled lost enthusiasm for mathematics, and others said that they now love math more than they did before.

Finances summary:

The income from student fees (some discounted) was \$95,026.

Our Epsilon Grant award was \$2,520.

Total MathILy income: \$97,546.

Administrative expenses (insurance, etc.) totaled approximately \$1,250.

Total wages (instructors, PRiME, Minion, Director) were approximately \$73,033.

Program expenses (supplies) were approximately \$4,783.

Total MathILy expenses: approximately \$79,066.

This year we benefitted from Thomas Hull's NSF standard grant, which supported three staff members (worth \$15,213). We were also fortunate to receive in-kind donations goods and services worth roughly \$7,280 in the form of software from Wolfram Research, and in-kind donations of volunteer time worth roughly \$195.

The net revenue of approximately \$18,480 arose from far lower than expected financial aid needs. We expect that when we return to in-person operations, there will be significant financial aid need because of the economic effects of the global pandemic, and this revenue will be applied toward that need.