



## MathILy 2020 Final Report

### Preface

MathILy was quite different this year—because of the global pandemic, the program was held synchronously online with 43 participants. We still managed to hold a program with excellent students who learned a lot of mathematics and a lot about how to think and speak and write mathematically.

### Program Preparations

#### Promotions

*Electronic:* Individual emails were sent to prior participants and promising applicants. MathILy continues to be listed on several high-traffic webpages, including MIT Admissions and AMS summer program listings. Web traffic varied from 8000–10000 hits/month, with about a third of the traffic from abroad. The second-most-visited page was Discrete Mathematics in the Real World, indicating that this is effective outreach.

*Print:* About 200 fliers with basic information on one side and a multi-part mathematics problem on the reverse were distributed in the Fall to national and local mathematics contests. Fliers were handed out by humans at February HMMT; later Spring events were cancelled because of the pandemic lockdown.

*Other Activities:* We held a {MathILy, MathILy-Er, MathILy-EST} Yearly Gather at the Joint Mathematics Meetings where the 36 participants, about 1/3 of whom were alums, team-solved a set of Jonah-Ostroff-designed puzzles that involved drawing paths through hexagonal grids. At HMMT February, sarah-marie gave a drop-in Mini-Event (on colorful polyhedral skeleta, 16 attendees) and was available to talk with students, parents, and coaches. In April, at the peak of school closings, she gave an Art of Problem Solving Math Jam (125–200 attendees at any time) on four numbers followed by a {MathILy, MathILy-Er} Q&A.

### Applications and Admissions

*Statistics:* We received 1058 Short Forms, 396 Not-as-Short Forms, 366 EARs, and 328 completed applications. We admitted 59 students; thus, our current admissions rate is roughly 18%. Sixteen students declined, but only one for a peer summer mathematics program.

*Demographics:* Applicants originated from at least 35 US states, 1 US territory, and 32 foreign countries (representing mostly Europe and Asia, but also including South America, Africa, and Oceania). The data in the following table was measured where possible and approximated otherwise; the final row reflects a post-program demographic survey given as part of assessment for the MathILy-EST NSF grant.

Percentage	Female	East Asian	South Asian	Latinx	Middle Eastern	Other of color
Short Forms	37%	35%	23%	2%	4%	2%
EARs	31%	42%	21%	2%	1%	1%
Attending	29%	38%	17%	10%	0%	5%

*Financial Aid:* We awarded \$6,350 in financial aid to MathILy participants, of which \$3,000 was provided by the AMS Epsilon Fund. Approximately 14% of admitted students applied for financial aid; we met the demonstrated need of all applicants.

## Personnel

*Academic:* Lead Instructors were Dr. Hannah Alpert (U. of British Columbia, Ph.D. MIT 2016), dr. sarah-marie belcastro (Math Staircase Inc., Ph.D. U. of Michigan 1997), Dr. Max Engelstein (U. of Minnesota, Ph.D. U. of Chicago 2016), Dr. Brian Freidin (U. British Columbia., Ph.D. Brown U. 2018), Dr. Nate Harman (U. of Chicago, Ph.D. MIT 2017), Dr. Thomas C. Hull (Western New England U., Ph.D. Univ. of Rhode Island 1997), Dr. Emily Peters (Loyola U. Chicago, Ph.D. UC Berkeley 2009) and Dr. Daniel Studenmund (Binghamton U., Ph.D. U. of Chicago 2014).

Apprentice Instructors were Emi Brawley (graduate student, UC Davis and MathILy 2013 alumn), Miles Dillon Edwards (AoPS Instructor, Indiana U. graduate), Nadav Kohen (software engineer, U. Iowa graduate, and MathILy 2015 alumn), Joshua Mundinger (graduate student at U. of Chicago and MathILy 2013 alumn), and Kye Shi (undergraduate at Harvey Mudd and MathILy 2015/2016 alumn).

The FRANK (Facilitates Rumination, Activating New Knowledge) was Frank Lu (undergraduate at Princeton and MathILy 2018 alumn).

Biographical information and prior experience are listed at [Dramatis Personae](#).

*Administrative:* The Director was dr. sarah-marie belcastro. The excellent {MathILy, MathILy-Er} Minion was Madison Stuart (Smith College B.A. 2006 in math and German; graduate work in information science at the University of Michigan). The Protector and Responder in the MathILy Environment (PRiME) was Cory Saunders (high-school teacher, Clarkson M.A.T. 2019, Haverford College B.A. in math 2017).

*Advisory Amalgam:* These individuals gave advice on academic and practical aspects of MathILy.

[Dr. Douglas J. Shaw](#), mathematics faculty at University of Northern Iowa

[Dr. Ruth Haas](#), mathematics faculty at University of Hawaii

[James Cocoros](#), mathematics faculty at Stuyvesant High School

[Dr. Dylan Shepardson](#), mathematics faculty at Mount Holyoke College

[Dr. Carol E. Fan](#), operations researcher (currently Director of Data Science at [Stellar Labs](#))

[Dan Zaharopol](#), Executive Director of [BEAM](#)

[Dr. James Tanton](#), mathematician, currently Ambassador for the [MAA](#)

[Dr. Joshua Greene](#), mathematics faculty at Boston College

[Dr. Emily Peters](#), mathematics faculty at Loyola University Chicago

[Wing L. Mui](#), Seattle-area artist and mathematics teacher

[Dr. Thomas Hull](#), mathematics faculty at Western New England University

[Dr. Josh Laison](#), mathematics faculty at Willamette University

## Student Demographics

*U.S. States represented by MathILy students, roughly from east to west:* Massachusetts, Connecticut, New York, New Jersey, Pennsylvania, Maryland, North Carolina, Ohio, Tennessee, Illinois, Missouri, Texas, Colorado, Arizona, Washington, California, and Hawaii.

*Countries outside of the United States represented by MathILy students:* China, Romania, Nigeria, Canada.

*Gender breakdown:* 12 females, 31 males.

*Ages:* Two 14-year olds; eleven 15-year olds; sixteen 16-year olds; thirteen 17-year olds; and one 18-year old.

*Academic backgrounds:* Two-thirds of the students had already taken calculus II or equivalent (and 16% had also taken multivariable calculus), five had taken linear algebra, and 20% had taken other more advanced courses. In contrast, 5% of the students had not yet taken precalculus. 29 students had attended summer mathematics programs before.

## What Happened at MathILy 2020?

### Academics

**Classes:** Classes met through Zoom for audio/video communication, with breakout rooms for group work; we also used Limnu shared online whiteboards, Google Drive for shared during-class typing by students without stylus input, Slack for during-class and outside-of-class communication, Google Classroom for submission/return of written work, Kami for commenting on written work, and our usual computer algebra systems Sage and *Mathematica* as well as optional LaTeX for typesetting.

Each weekday we had 4 hours of morning class, 1–1.5 hours of Daily Gather, and 3 hours of evening class, for at least 8 contact hours per day. The general Saturday template consisted of 4 hours of morning class and 2 hours of afternoon Life Seminar, and we abstained from formally scheduled activities on Sundays so students would have a full non-Zoom day.

The basic curricular structure was two weeks of core curriculum, called Root Class (after the root of a graph theoretic tree, and after the idea that the material strengthens student grounding much as the roots of a tree do), followed by one week of short topical classes, called Week of Chaos, followed by two weeks of focused-topic classes, called Branch Class (after branches of mathematics, and after the idea that tree branches grow from a strong trunk nourished by roots).

*Root Class:* There were three Root classes, each with 14 or 15 students, one with {sarah-marie, Emi, Frank}, one with {Daniel+Tom, Josh, Kye-then-Cory}, and one with {Brian, Nadav, Cory-then-Kye}. Our core curriculum consisted of linear and affine algebra and geometry (including equations and intersections of hyperplanes, span, linear independence, transformations, and dimension), combinatorics, graph theory, definition and examples of groups, isomorphism for various categories, probability spaces and expected value, and basic cardinality. Of course, all of this material was treated with full proofs given by the students.

*Week of Chaos:* Students indicated which of 58 potential topics they would be excited to learn about, from which instructors decided on a list of 27 classes offered. These were: Keeping Secrets, Party Parrot Party Games Parity Project (game theory),  $x_1, ah\ ah\ ah!$   $x_2, ah\ ah\ ah!$   $x_1x_2x_3, ah\ ah\ ah!$  (combinatorial nullstellensatz), Moar Linear Algebra, Tunnels Mazes and Escape Rooms (finite-state automata), Algebraic Geometry, How to Study TMS Syndrome (Markov chains), Color Theory (graph coloring), Math Saves the World: An Introduction to Infectious Disease Modeling, To Infinity! (And Beyond?), Spaces of Spaces (moduli spaces of polygons), Representation Theory or the Symmetries of Everything, Planning Your Dynasty (algorithms), Why We Can't Have Nice Things (voting theory), Strange Geometries, Knot Theory!, Math 'n' Music, SuperCats and Slammin' Hats (groups, rings, and morphisms), Cats (isoperimetric problems), Fibonacci Identities, When is a Line not a Line? (fractals), Numbers That Aren't Real ( $p$ -adics), Math Saves the World: Combinatorial Optimization, Generatingfunctionology, Mouse Torture (sorting algorithms), Realer than Real (surreal numbers), and Holograms? Holodecks? (complex analysis). Student preferences guided placement of each student into 5 classes. More than half of these classes used specific material from the Root curriculum, approximately 1/3 benefitted substantially from students' knowledge or understanding of linear algebra, and approximately 1/4 used computer algebra systems in class.

*Branch Classes:* We offered three Branch classes, one on convex geometry and polytopes (sarah-marie and Josh), one on chip-firing games (Brian and Emi with Frank), and one on the mathematics of paperfolding (Tom and Kye with Cory). All three Branch classes used linear algebra and two of them introduced applications.

*Pedagogy:* All classes were conducted using inquiry-based learning, with the bulk of the time spent with students working in groups or presenting their insights to each other and a much smaller amount of time used by faculty conducting discussion from the shared whiteboards.

*Feedback:* Students received feedback in multiple ways. During class, they received instant verbal feedback on the correctness of their mathematical ideas, and also on use of notation, language, and presentation style. Likewise, students received daily written feedback on their mathematical writing. Near the end of Root and Branch classes, each student was asked to write an introspective self-evaluation. The self-evaluations were discussed by the student's instructors, and the instructors then held a 5–20 minute meeting with each student to give overall feedback on the student's progress at MathILy and advice for the future.

*Interactions with MathILy-EST:* MathILy-EST participants intermixed with MathILy students for group work in Daily Gathers throughout the program. Each MathILy-EST participant took a Week of Chaos class and instructors thought this was a beneficial experience for all parties.

**Daily Gathers:** Each instructor gave a Daily Gather, as did the MathILy-EST research group. The Daily Gather timeslot was also used to show Math Movies once per week. These included expository films made by the Mathematical Association for America, films from the National Film Board of Canada, and narrated animations made by individual mathematicians or research groups. The remainder of the Daily Gathers were given by visiting mathematicians.

## Extracurriculars

*Life Seminars:* There were four weekend Life Seminars offered. The first was on practical matters such as how to address faculty (in person or by email) and impostor syndrome. The second Life Seminar was on careers for people with mathematical science training, for which we had guests from industry. The third Life Seminar was about preparing for Branch and when to start research. The final Life Seminar was on how to choose colleges to which to apply, and included advice from the MathILy-EST participants as well as an exchange of information about instructor experiences at colleges and universities.

*Other all-program activities:* Before the opening meeting we had four hours of structured socializing involving Two Truths and a Lie and Haxball. At the end of the first Life Seminar, we played telephone pictionary in teams using Drawception. The program went by train to Philadelphia for an hour on Saturday morning right after Week of Chaos—using Google Earth and virtual tours. The following day about half of the program celebrated National Ice Cream Day over Zoom (though perhaps half of the attendees actually had ice cream).

*Non-program-wide activities:* Students organized daily afternoon games in the Common Zoom. A staff member hosted a Sunday-morning breakfast Zoom to which many participants brought the traditional chocolate-chip muffins. Most nights there were optional graphic-novel Bedtime Stories. A few students used the Zoom annotation features to draw on top of math movies (e.g. making a border of small hearts in time to the soundtrack). Many classes had hilarious Limnu pin names, with themes changing by the class and by the day. The Slack party parrot emoji caught on so much with students that party parrots were featured on our t-shirt and used as the theme of a Daily Gather, a Week of Chaos class, and a Branch

class. The Unstable Wifi Drill included an arm-waving dance to indicate frozen video, which was done throughout the program by all parties.

## Administrative matters

*Student early/late time zones:* We held activities in Central time because it was closest to average; this resulted in four students in very different time zones ranging from 5 hours early to 13 hours later.

*Technology selection and preparation:* We made extensive use of nine different pieces of software this summer, only three of which (LaTeX, Sage, and *Mathematica*) are ordinarily used at MathILy in person. Four others were clear choices (Zoom, Google Drive, Google Classroom, and Slack). However, much investigation was needed to find appropriate PDF annotation software (Kami) and we compared at least eighteen different shared online whiteboards before settling on Limnu.

We also wrote documentation for each software, in separate versions for staff and students: installation instructions, MathILy-specific protocols, and tips for setting up and using the software. Staff then did a tech check with each student to uncover and resolve unexpected technological issues.

## Post-Processing

*Post-program staff meeting:* After the official end of the program, the staff convened to evaluate various aspects of the program and to discuss how we could improve the workings of MathILy in future. Of course, we discussed what we should change were we to be online in a future year. There was much brainstorming of tweaks that might collectively reduce the significant additional time and energy required for teaching online as well as plans for restructuring social activities. Aspects we plan to bring forward to in-person MathILy include doing pre-program tech checks, using Slack, randomizing seating in Daily Gather (by using seating chits) and moving some post-program meetings to Zoom.

*Impact:* As usual, many students commented that they learned about areas of math of which they hadn't even dreamed before MathILy, and that they grew substantially in their proofwriting prowess. Several commented specifically on how much more effective the online format was, both academically and interpersonally, than they had expected.

### *Finances summary:*

The income from student fees (some discounted) was \$86,100.

Our Epsilon Grant award was \$3,000.

Total MathILy income: \$89,100.

Administrative expenses (insurance, fliers, etc.) totaled approximately \$1,872.

Total wages (instructors, PRiME, Minion, Director) were approximately \$55,624.

Program expenses (supplies) were approximately \$2,959.

Total MathILy expenses: approximately \$60,455.

This year we benefitted from Thomas Hull's NSF standard grant, which paid for three staff members' wages (worth \$11,667). We were also fortunate to receive in-kind donations goods and services worth roughly \$7,326, primarily in the form of software from Wolfram Research, and in-kind donations of volunteer time worth roughly \$5,467.

The net revenue of approximately \$28,645 arose from many more students participating in the program than feared, along with much lower than expected financial aid needs. We expect that when we return to in-person operations, there will be significant financial aid need because of the economic effects of the global pandemic, and this revenue will be applied toward that need.