



## MathILy 2019 Final Report

### Preface

MathILy was slightly smaller this year than last (but comparable in size to the prior two years). As usual, we held a program with excellent students who learned a lot of mathematics and a lot about how to think and speak and write mathematically.

### Program Preparations

#### Promotions

*Electronic:* Emails were sent to promising applicants from 2018 and to {MathILy, MathILy-Er} 2018 participants. MathILy continues to be listed on several high-traffic webpages, including the summer program listing pages hosted by the AMS and MIT Admissions. Web traffic varied from 2000–3600 hits/week, with about a quarter of the traffic from abroad.

*Print:* About 400 fliers (about 1/10 our usual number) with basic information on one side and a multi-part mathematics problem on the reverse were distributed to national and local mathematics contests. Fliers were handed out by humans at HMMT (November and February) and ARML Penn State.

*Other Activities:* We held a {MathILy, MathILy-Er} Yearly Gather at the Joint Mathematics Meetings where the 24 participants, about half of whom were recent alumns, team-solved a multi-part polyomino placement puzzle. At HMMT February, sarah-marie gave a Mini-Event (on embedding graphs on the torus, 21 attendees) and at ARML Penn State she gave one of two Friday Night Lectures (on frieze patterns, 257 attendees; the other was the Arnold Ross lecture, by Bjorn Poonen). At both events sarah-marie met with potential/current applicants, alumns, coaches and a few parents. She also offered a very successful Art of Problem Solving Math Jam (80–100 attendees at any time) on the  $n$ -cube that included a {MathILy, MathILy-Er} Q&A.

### Applications and Admissions

*Statistics:* We received 958 Short Forms, 365 Not-as-Short Forms, 337 EARs, and 306 completed applications. We admitted 56 students; our current admissions rate is roughly 18%, a record low. Nine students declined for other summer mathematics programs.

*Demographics:* Applicants originated from at least 39 US states and 29 foreign countries (representing mostly North America, Europe and Asia, but also including South America and Africa).

The data in the following table was measured where possible and approximated otherwise.

Percentage	Female	East Asian	South Asian	Latinx	Middle Eastern	Other of color
Short Forms	37%	37%	10%	3%	3%	2%
EARs	35%	39%	12%	3%	1%	2%
Attending	29%	48%	7%	5%	3%	0%

*Financial Aid:* We awarded \$15,900 in financial aid to MathILy participants, of which \$5,000 was provided by the AMS Epsilon Fund. Approximately 18% of admitted students applied for financial aid; we met the level of demonstrated need for all applicants.

## Personnel

*Academic:* Lead Instructors were Dr. Hannah Alpert (Ohio State U., Ph.D. MIT 2016), dr. sarah-marie belcastro (Smith College Research Associate, Ph.D. U. of Michigan 1997), Dr. Max Engelstein (U. of Minnesota, Ph.D. U. of Chicago 2016), Dr. Brian Freidin (Brown U., Ph.D. Brown U. 2018), and Dr. Thomas C. Hull (Western New England U., Ph.D. Univ. of Rhode Island 1997).

Apprentice Instructors were Connor Ahlback (graduate student, U. Washington), Shamil Asgarli (graduate student, Brown U.), Emi Brawley (graduate student, UC Davis and MathILy 2013 alumn), Connor Halleck-Dube (Mathematical Tripos Part III), Nadav Kohen (software engineer, U. Iowa graduate, and MathILy 2015 alumn), Joshua Munding (graduate student at U. of Chicago and MathILy 2013 alumn), and Corrine Yap (graduate student, Rutgers U.).

Biographical information and prior experience are listed at [Dramatis Personae](#).

*Administrative:* The Director was dr. sarah-marie belcastro. The excellent {MathILy, MathILy-Er} Minion was Madison Stuart (Smith College B.A. 2006 in math and German; graduate work in information science at the University of Michigan). The complementary Protectors and Responders in the MathILy Environment (PRiMEs) were Mary Turner (St. John's College math major) and Alex Nolte (Tufts graduating senior).

*Advisory Amalgam:* These individuals gave advice on academic and practical aspects of MathILy.

[Dr. Douglas J. Shaw](#), mathematics faculty at University of Northern Iowa

[Dr. Ruth Haas](#), mathematics faculty at University of Hawaii

[James Cocoros](#), mathematics faculty at Stuyvesant High School

[Dr. Dylan Shepardson](#), mathematics faculty at Mount Holyoke College

[Dr. Carol E. Fan](#), operations researcher (currently Director of Data Science at [Stellar Labs](#))

[Dan Zaharopol](#), Executive Director of [BEAM](#)

[Dr. James Tanton](#), mathematician, currently Ambassador for the [MAA](#)

[Dr. Joshua Greene](#), mathematics faculty at Boston College

[Dr. Emily Peters](#), mathematics faculty at Loyola University Chicago

[Wing L. Mui](#), mathematics faculty at the Overlake School

[Dr. Thomas Hull](#), mathematics faculty at Western New England University

[Dr. Josh Laison](#), mathematics faculty at Willamette University

## Student Demographics

*States represented by MathILy students:* Maine, Massachusetts, New York, New Jersey, Maryland, North Carolina, Florida, Kentucky, Missouri, Minnesota, Texas, Arizona, Idaho, and California.

*Countries outside of the U.S. represented by MathILy students:* South Korea, China, Poland, and Brazil.

*Gender breakdown:* 11 females, 27 males.

*Ages:* Two 14-year olds; nine 15-year olds; fifteen 16-year olds; eight 17-year olds; and four 18-year olds.

*Academic backgrounds:* A bit over half of the students had already taken calculus II or equivalent (and 13% had also taken multivariable calculus), and two had taken linear algebra. In contrast, 10% of the students had not yet taken precalculus. Twenty-five students had attended summer mathematics programs before.

## What Happened at MathILy 2019?

### Academics

**Classes:** Each weekday we had 4 hours of morning class, 1–1.5 hours of Daily Gather, and 3 hours of evening class, for at least 8 contact hours per day (not counting mathematical conversations outside of class). Weekends were, as always, a bit idiosyncratic, but the general Saturday template consisted of 4 hours of morning class and 2 hours of afternoon Life Seminar.

The basic curricular structure was two weeks of core curriculum, called Root Class (after the root of a graph theoretic tree, and after the idea that the material strengthens student grounding much as the roots of a tree do), followed by one week of short topical classes, called Week of Chaos, followed by two weeks of focused-topic classes, called Branch Class (after branches of mathematics, and after the idea that tree branches grow from a strong trunk nourished by roots).

*Root Class:* There were three Root classes, each with 12 or 13 students, one taught by {sarah-marie, Nadav}, one taught by {Hannah, Connor, Shaiml}, and one taught by {Brian, Emi}. Our core curriculum consisted of linear and affine algebra and geometry (including equations and intersections of hyperplanes, span, linear independence, transformations, and dimension), combinatorics, graph theory, definition and examples of groups, isomorphism for various categories, probability spaces and expected value, and basic cardinality. Of course, all of this material was treated with full proofs given by the students.

*Week of Chaos:* Students indicated which of 46 potential topics they would be excited to learn about, from which instructors decided on a list of 27 classes offered. These were: Game Theory/Nash Equilibrium, Combinatorial Nullstellensatz, Moar (Linear) Algebra, Simple Computers (finite-state automata), Origametry, Knotty Questions, Driven to Abstraction (algebraic subobjects and morphisms), Quadratic Reciprocity, I love to count and I cannot lie (combinatorics), Cookie Sets (the Mandelbrot set), What shape is best? (isoperimetric problems), Fibentities, Random Woks, Bagel Integration (Lebesgue integration), Vanishing Sets of Polynomials in Geometry and Combinatorics, When is a Line not a Line? (fractals), The penumbers ( $p$ -adics), Ramsey theory, Math Saves the World: Combinatorial Optimization, Infrmtn Thry, Magic Functions (generating functions), Fancy Note-Taking Diagrams (sorting algorithms), Human Transmutation (the probabilistic method), Big Finite Diff (finite difference calculus), Surreal Numbers: A Play in Five Acts, Loops on Loops on Loops (homotopy theory), and Holograms? Holodecks? (complex analysis). Student preferences guided placement of each student into 5 classes. Approximately half of these classes used specific material from the Root curriculum, and approximately 1/4 benefitted substantially from students' background in linear algebra.

*Branch Classes:* We offered three Branch classes, one on topological graph theory (sarah-marie and Shamil), one on chip-firing games (Brian and Connor), and a completely new one on persistent homology (Hannah and Corrine). All three Branch classes used linear algebra, and all three of them introduced real-world applications.

*Pedagogy:* All classes were conducted using inquiry-based learning, with the bulk of the time spent with students working in groups or presenting their insights to each other and a much smaller amount of time used by faculty conducting discussion from the board.

*Feedback:* Students received feedback in multiple ways. They received instant verbal feedback on the correctness of their mathematical ideas during class, as well as feedback on use of notation, language, and presentation style. Likewise, students received daily written feedback on their mathematical writing. Near the end of Root and Branch classes, each student was asked to write an introspective self-evaluation. The self-evaluations were

discussed by the student's instructors, and the instructors then held a 5–20 minute meeting with each student to give overall feedback on the student's progress at MathILy and advice for the future.

*Interactions with MathILy-EST:* The college students were somewhat shy about interacting mathematically with MathILy. However, most of them took a couple of Week of Chaos courses and instructors thought this was an interesting experience for all parties.

**Daily Gathers:** Each instructor gave a Daily Gather, as did the MathILy-EST research group. The Daily Gather timeslot was also used to show Math Movies once per week. These included expository films made by the Mathematical Association for America, films from the National Film Board of Canada, and narrated animations made by individual mathematicians or research groups. The remainder of the Daily Gathers were given by visiting mathematicians, both local (Franklin&Marshall, Lafayette, and Villanova) and from California, Illinois, and New York.

### **Extracurriculars**

*Life Seminars:* There were four weekend Life Seminars offered. The first was on practical matters such as how to address faculty in person or by email, how to properly write with chalk, and impostor syndrome. The second Life Seminar was on careers for people with mathematical science training. The third Life Seminar focused on preparing for Branch. The final Life Seminar was on how to choose colleges to which to apply, and included advice from the MathILy-EST participants as well as an exchange of information about instructor experiences at several colleges and universities.

*Other all-program activities:* At the end of the first week, we walked over to Haverford College, had a mini Daily Gather in their math lounge, and returned to Bryn Mawr to play a rousing game of telephone pictionary. The program went by train to Philadelphia for the entire Saturday right after Week of Chaos. The following day the program was treated to ice cream at a local shop for National Ice Cream Day.

*Non-program-wide activities:* There was generally a substantial group of students playing games in the living room, a smaller group in an air-conditioned lounge, and in reasonable weather, a group of students playing sports outdoors. Starting in Week of Chaos, MathILy-EST participants started playing sports and socializing in some other ways with MathILy participants. Many athletic activities were organized by a student who posted notes on the dorm door declaring times and activities for the day. These included frisbee, soccer, football, swimming, whiffleball, and capture the flag. A group of students ran regularly in the mornings. After some sports activities students did organized stretching, and frequently a group of students gathered in the main hallway to do “the MathILy workout” of calisthenics. Card games of various sorts were popular, with Avalon (a hybrid social-engineering and card game) most popular. A few students sang together as they walked from place to place. Several students practiced instruments, and from time to time the piano in the living room was used. Most nights there were optional Bedtime Stories. On the last night of the program, we brought lots of puzzles to the living room and many of the students played with them late into the night.

### **Administrative matters**

*Campus Location:* Bryn Mawr is a safe and tree-filled suburb of Philadelphia, located on a major train line into the city. Bryn Mawr College is a few blocks away from the Bryn Mawr train station, and also a few blocks away from a major street that parallels the train line and has lots of shops, libraries, post offices, etc. This location was convenient for our purposes—many desired destinations were a short walk away or required a straightforward and quick drive. Students were able to depart campus on foot in groups to go out for dinner or shopping. It was also straightforward to take the program into the city, and use of the train made our travel flexible.

*Facilities at Bryn Mawr:* Again, everyone liked the facilities and the campus as a whole. We were assigned our own dormitory this year. All students had air-conditioned rooms; almost all had single rooms, and some shared one- or two-room doubles. There were also two air-conditioned lounges, a large living room, and a large entry way for common space. In addition to our usual classrooms in Park Science Center, we had access to an awesome glassed-in full classroom. With the addition of MathILy-EST to our community, we outgrew our former Daily Gather room and moved into the large tiered lecture room next to our newest classroom. As before, every (early) morning janitorial staff cleaned the chalkboards and tidied the rooms.

## Post-Processing

*Post-program staff meeting:* After the students left, the staff convened to evaluate various aspects of the program and to discuss how we could improve the workings of MathILy in future. We identified some tweaks to make and some projects to do. For tweaks, we plan to change the frequency with which we watch a historical video during teacher training, change the file structure of the Root problems database, make an acronym appendix for the PRiME Guide, and require some structured interactions between MathILy-EST and MathILy participants. For projects, we plan to create an analysis sequence of alumni worksheets and rethink the one-shot alumni worksheets.

*Impact:* As usual, many students commented

- that they learned about areas of math of which they hadn't even dreamed before MathILy
- that they are even more passionate about mathematics.

Additionally, several students said that they learned the value (and skill!) of asking questions, and many commented that they grew substantially in their proofwriting prowess.

*Finances summary:*

The income from student fees (some discounted) was \$160,765.

Our Epsilon Grant award was \$5,000.

Total MathILy income: \$165,765.

Administrative expenses (insurance, fliers, etc.) totaled approximately \$2,279.

Total wages (instructors, PRiME, Minion, Director) were approximately \$59,844.

Travel costs (Daily Gather speakers, instructors) were approximately \$2,343.

Program expenses (supplies, program outings) were approximately \$2,952.

Site (Bryn Mawr College) charges, including housing, meals, and duplications were \$105,044.

Total MathILy expenses: approximately \$172,462.

The net loss of approximately \$6,700 arose from financial aid needs that exceeded program income, despite having some federally supported instructors.

An externally administered travel grant, together with in-kind donations by refusal of travel reimbursements saved \$2,104. We were also fortunate to receive in-kind donations goods and services worth roughly \$14,213, primarily in the form of software from Wolfram Research, and in-kind donations of volunteer time worth roughly \$9,925.