



MathILy-EST 2024 Final Report

Preface

The MathILy-EST Research Experience for Undergraduates (REU) was created to serve college-age students that are early in their college career (i.e., an emphasis on freshmen, but with consideration for sophomores and even graduating high-school seniors). Also, this REU runs in parallel to the 5-week MathILy program for talented high-school students. Both take place at Bryn Mawr College and all the students and staff share the same dorm building.

Program Preparations

Promotions

Emails: Notes advertising MathILy-EST were sent to multiple email lists in the professional math community, and to (an updated list of) contacts at Historically Black Colleges and Universities, Minority Serving Institutions, the Hispanic Association of Colleges and Universities, Leadership Alliance coordinators, and to contacts at McNair programs and Black and Latinx student centers.

Fliers: No MathILy-EST-specific fliers were handed out, though 870 {MathILy, MathILy-Er} fliers that mention MathILy-EST were handed out at HMMT and sent to seven high school math contests.

Webpages and links: MathILy-EST has its own webpages, and is listed on the NSF-REU pages (of course) as well as the AMS Opportunities pages, the Institute for Broadening Participation's pathwaystoscience.org, the Math Alliance website, and the Art of Problem Solving's wiki. There are also several online lists of math REUs that include MathILy-EST.

Webpage hits: There were about 15,300 impressions for the mathilyest/index.html page and 5,300 for the mathilyest/facts.html page over the 2023–2024 season.

Other Activities: In late February sarah-marie offered an Art of Problem Solving Math Jam on REUs in general and MathILy-EST in particular, which was mainly a Q&A. There were about 35 people in the room at most times, and about 200 came/went over the 90 minutes.

Applications and Admissions

Demographics: There were 128 completed applications for the 9 REU slots. Applicants originated from 32 US states. Of course, some states were over-represented in that number, with at least 22 from PA, 17 from NY, 17 from CA, and 12 students from MA schools.

In terms of the demographics during the stages of making decisions, we have the following percentages:

| Stage in application | Female | NB/Trans | Asian-American | Black | Latinx | SLAC |
|----------------------|--------|----------|----------------|-------|--------|------|
| All applicants | 22% | 7% | 32% | 5% | 4% | 24% |
| Long list (33) | 51% | 9% | 24% | 9% | 12% | 18% |
| Short list (16) | 38% | 13% | 19% | 6% | 6% | 18% |
| Accepted | 44% | 11% | 33% | 11% | 11% | 0% |

All invited students accepted within 24 hours (some within only a few hours). A tenth, international student was also invited to participate, although that student was not funded by the NSF grant.

Personnel

Administrative: The MathILy-EST 2024 Director was Dr. Thomas Hull (Associate Professor of Applied Mathematics, Franklin and Marshall College). Natalya (Natasha) Ter-Saakov (graduate student, Rutgers University) was the Graduate Research Apprentice menTor at MathILy-EST (GReAT-EST). The PI on the NSF grant was sarah-marie belcastro (President of Mathematical Staircase, Inc.). The {MathILy, MathILy-Er, MathILy-EST} Minion was Madison Stuart.

Senior Personnel: These individuals gave advice on the construction of MathILy-EST and the NSF proposals for the grants that fund the program.

Hannah Alpert, former mathematics faculty at Auburn University (MathILy-EST director 2020)

Max Engelstein, mathematics faculty at University of Minnesota (MathILy-EST director 2021)

Brian Freidin, mathematics faculty affiliated with Auburn University, teaches at University of San Diego (MathILy-EST director 2023)

Nate Harman, mathematics faculty at University of Georgia (MathILy-EST director 2022)

Thomas Hull, mathematics faculty at Franklin and Marshal College (MathILy-EST director 2019)

Emily Peters, mathematics faculty at Loyola University Chicago

What Happened at MathILy-EST 2024?

Academics/Research

The research topic was the combinatorial geometry of origami, paper folding. The students formed three groups working on the stamp-folding problem, origami flip graphs, and rigid foldings.

Reading: A week before the program started, the Director sent pairs of -EST students research papers to read that covered background on origami math. Each pair made a Coauthor post summarizing their paper and gave a presentation to the other students on it during the first week.

Mathematical Explorations: The stamp-folding group looked at an old unsolved problem: enumerating the number of ways a $1 \times n$ strip of stamps can be folded, including different layer orderings of the paper. A new twist they took on this problem was to fix a given mountain-valley (MV) assignment and then try enumeration. They wrote efficient code to do this and discovered many patterns. For example, the

$(M^2V^2)^k$ MV assignment, which is $MMVV$ repeated k times, has the $(k + 1)$ st Catalan number squared number of ways to fold. The students developed proofs for many of the patterns they discovered as well as found an upper bound on the number of ways a MV assignment can fold.

The origami flip graph group focused on the classic Miura-ori crease pattern. They discovered that the $2 \times n$ Miura-ori's origami flip graph has exactly four vertices of degree 2, and that these degree-2 vertices are at maximal distance from each other in the graph. In addition to proving these structural features of the $2 \times n$ Miura-ori origami flip graph, they also developed a recursive transfer matrix for enumerating the vertices in the $m \times n$ case.

The rigid-folding group created two different geometric ways to interpret the folding angle equations for a rigidly folding flat-foldable degree-4 vertex. They also devised a recursive algorithm for describing the rigid origami configuration space of general degree- n vertex, and used discrete Gaussian curvature to prove rigorously that the parallel-pleat transform of Hull and Tachi preserves kinematics. The latter was used by the students to construct a rigidly folding parallel-pleat version of a degree-5 vertex, which had never been done before.

Professional Development

Writing: The MathILy-EST students each wrote daily on Coauthor to record their ideas, progress, and pitfalls. Every Ever-EST seminar had an official notetaker to write a Coauthor post on the event. In addition, the students were given writing assignments and feedback by the Director and the GReAT-EST. Each group prepared a draft of the final papers using Overleaf, with the eventual goal of having it placed on the arXiv and submitted for publication. Participants were also trained on writing CVs and tasked with creating their own professional CVs.

Presentations: The MathILy-EST 2024 program required the participants to make many presentations. Every time that MathILy had a visitor to give a Daily Gather, the visitor also met with the MathILy-EST students, whereupon the students presented their research topics and work-to-date to the visitor. During Week 3 the program was visited by the Villanova-Puerto Rico Research Retreat (VPR³), directed by Alexander Diaz-Lopez of Villanova, and the two programs presented their in-progress research work to each other. During Week 6 MathILy-EST visited Dr. Cynthia Sung's robotics lab at the University of Pennsylvania to meet her summer research students. The -EST students gave their presentations to a dozen or so people from Dr. Sung's lab, and her students presented the robotics devices that they were working on to the -EST students. During the last week of MathILy, the MathILy-EST students gave a Daily Gather, titled "Stamps, Lamp Cities, and gEometry (SPACE!!)," on their research work to the MathILy students. The -EST students this year were very enthusiastic about giving presentations, and by the end of the program all of the groups had submitted their talk abstracts for the 2025 Joint Mathematics Meetings in Seattle, WA!

Software: Throughout MathILy-EST the participants used various software packages to aid in their research explorations. They used *Mathematica*, Sage/Python, and Geogebra, extensively.

Other professional development: The Director and the GReAT-EST also provided training to the MathILy-EST students on how to approach research and not get discouraged, giving good math presentations, looking up articles on MathSciNet or other databases, examining ethics in mathematics and academic culture, and other resources for undergraduates in mathematics. There was a graduate

school panel for the MathILy-EST students that included the current MathILy AIs, PRiME FACToRs, and Tom and Natasha.

Social Activities: The MathILy-EST 2024 participants formed a very tight, enthusiastic, and collegial cohort. Activities included karaoke, game-playing, origami, and a trip to see the Rocky Horror Picture Show.

Administrative matters

Facilities at Bryn Mawr: The physical environment that Bryn Mawr provided for the MathILy-EST students was excellent. Three modern “fish bowl” style conference rooms in the recently renovated section of the Park Sciences Building were provided for exclusive use by the REU students. These rooms each had floor-to-ceiling blackboards and a wide-screen TV for projecting computers, and one also had a whiteboard.

Post-Processing

Post-program senior personnel meeting: After the end of the program, there was a meeting to discuss the program and to plan for next year. Dr. Joshua Mundinger will direct the REU in 2025.

Impact: Students rated all aspects of their MathILy-EST experience as very or somewhat valuable (except for one “not useful” rating in 1/3 of the categories and two “not useful” ratings of MathILy Life Seminar), and all nine participants stated that participation in MathILy-EST has positively influenced their career path or career. Every student visited a MathILy class, and all found it an interesting experience. Almost all MathILy-EST participants reported socializing with MathILy students outside of their classes, and all found it fun and interesting, and most found it valuable. Over 95% of the MathILy students found MathILy-EST somewhat or very valuable to their MathILy experience in general and on each specific aspect queried.

Finances summary:

The income from grant NSF DMS-2149647 was \$111,893.

Total MathILy-EST income: \$111,893.

Total stipends (director, participants) were \$55,570.

Total wages (MathILy director and Minion) were \$1420.

Total non-wage employee expenses were \$45.

Program expenses (t-shirts, pens, stickers) were approximately \$259.

Travel expenses for participants were \$9,669.

Site expenses from Bryn Mawr were \$45,539.

Total MathILy-EST expenses: \$112,502.

The difference between income and expenses arises mainly from asynchrony between paying expenses and requesting reimbursement, and from non-NSF-reimbursable program expenses. Note that the above includes travel expenses from the Joint Mathematics Meetings for the 2023 MathILy-EST cohort.